Optimal Source Coding with Signal Transfer Function Constraints

Milan Stefan Derpich Musa

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School of Electrical Engineering
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Callaghan, NSW 2308
Australia

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I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

Milan S. Derpich

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Abstract

This thesis presents results on optimal coding and decoding of discrete-time stochastic signals, in the sense of minimizing a distortion metric subject to a constraint on the bit-rate and on the signal transfer function from source to reconstruction.

The first (preliminary) contribution of this thesis is the introduction of new distortion metric that extends the *mean squared error* (MSE) criterion. We give this extension the name *Weighted-Correlation MSE* (WCMSE), and use it as the distortion metric throughout the thesis. The WCMSE is a *weighted* sum of two components of the MSE: the variance of the error component uncorrelated to the source, on the one hand, and the remainder of the MSE, on the other. The WCMSE can take account of signal transfer function constraints by assigning a larger weight to deviations from a target signal transfer function than to source-uncorrelated distortion.

Within this framework, the second contribution is the solution of a family of feedback quantizer design problems for wide sense stationary sources using an additive noise model for quantization errors. These associated problems consist of finding the frequency response of the filters deployed around a scalar quantizer that minimize the WCMSE for a fixed quantizer *signal-to-(granular)-noise ratio* (SNR). This general structure, which incorporates pre-, post-, and feedback filters, includes as special cases well known source coding schemes such as *pulse coded modulation* (PCM), *Differential Pulse-Coded Modulation* (DPCM), Sigma Delta ($\Sigma\Delta$) converters, and noise-shaping coders. The optimal frequency response of each of the filters in this architecture is found for each possible subset of the remaining filters being given and fixed. These results are then applied to oversampled feedback quantization. In particular, it is shown that, within the linear model used, and for a fixed quantizer SNR, the MSE decays exponentially with oversampling ratio, provided optimal filters are used at each oversampling ratio. If a subtractively dithered quantizer is utilized, then the noise model is exact, and the SNR constraint can be directly related to the bit-rate if entropy coding is used, regardless of the number of quantization levels. On the other hand, in the case of fixed-rate quantization, the SNR is related to the number of quantization levels, and hence to the bit-rate, when overload errors are negligible. It is shown that, for sources with

unbounded support, the latter condition is violated for sufficiently large oversampling ratios. By deriving an upper bound on the contribution of overload errors to the total WCMSE, a lower bound for the decay rate of the WCMSE as a function of the oversampling ratio is found for fixed-rate quantization of sources with finite or infinite support.

The third main contribution of the thesis is the introduction of the *rate-distortion function* (RDF) when WCMSE is the distortion metric, denoted by WCMSE-RDF. We provide a complete characterization for Gaussian sources. The resulting WCMSE-RDF yields, as special cases, Shannon's RDF, as well as the recently introduced *RDF for source-uncorrelated distortions* (RDF-SUD). For cases where only source-uncorrelated distortion is allowed, the RDF-SUD is extended to include the possibility of linear-time invariant feedback between reconstructed signal and coder input. It is also shown that feedback quantization schemes can achieve a bit-rate only 0.254 bits/sample above this RDF by using the same filters that minimize the reconstruction MSE for a quantizer-SNR constraint.

The fourth main contribution of this thesis is to provide a set of conditions under which knowledge of a realization of the RDF can be used directly to solve encoder-decoder design optimization problems. This result has direct implications in the design of subband coders with feedback, as well as in the design of encoder-decoder pairs for applications such as networked control.

As the fifth main contribution of this thesis, the RDF-SUD is utilized to show that, for Gaussian stationary sources with memory and MSE distortion criterion, an upper bound on the information-theoretic causal RDF can be obtained by means of an iterative numerical procedure, at all rates. This bound is tighter than 0.5 bits/sample. Moreover, if there exists a realization of the causal RDF in which the reconstruction error is jointly stationary with the source, then the bound obtained coincides with the causal RDF. The iterative procedure proposed here to obtain $\overline{R_c^{it}}(D)$ also yields a characterization of the filters in a scalar feedback quantizer having an operational rate that exceeds the bound by less than 0.254 bits/sample. This constitutes an upper bound on the optimal performance theoretically attainable by any causal source coder for stationary Gaussian sources under the MSE distortion criterion.

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